Journal of Applied Mechanics and Technical Physics, Vol. 47, No. 1, pp. 54–60, 2006

EFFECT OF THE NONMONOTONIC TEMPERATURE DEPENDENCE OF WATER DENSITY ON THE DECAY OF AN INITIAL DENSITY DISCONTINUITY

V. I. Bukreev

UDC 532.5.013.4

The effect of the nonmonotonic temperature dependence of water density on gravity current flow after removal of the shield which separated warm and cold water was studied experimentally. If the temperature of water of maximum density was in the interval between the initial temperatures on the different sides of the shield, Rayleigh–Taylor instability developed along with shear instability under normal conditions.

Key words: *temperature of water of maximum density, density flow, Rayleigh–Taylor instability.*

One of the anomalous properties of water is that the temperature dependence of water density is nonmonotonic. The water density is maximum at a temperature of 4°C (at atmospheric pressure). If water masses at temperatures above and below 4°C come into contact, a layer forms whose density is higher than that of the ambient liquid and Rayleigh–Taylor instability develops. Such conditions occur, for example, in spring as a result of fluvial runoff and the more intense heating of shallow-water areas of pools, and in autumn as a result the snowfall in a warmer opened pool or rainfall in a colder open pool. The seasonal thermal bar has been the subject of extensive full-scale studies and numerical calculations [1, 2]. Under laboratory conditions, the hydrophysical effects due to the anomalous temperature dependence of water density have been studied using as an example the propagation of flat jet [3, 4] and circular jet [5].

The present paper gives some results of experiments performed as follows. A rectangular tank with a horizontal bottom was divided into two parts by a vertical shield. The left part was filled with water at a temperature T_1, and the right part with water at a temperature T_2. The water depth H was identical in both parts. At the time $t = 0$, the shield was removed vertically upward. The gravity current arising in this case has been the subject of extensive research. Here we note a monography [6] and a theoretical paper [7]. In most of the previous experiments, the initial density difference was produced by the addition of salt. In the present work, the density was varied by changing the temperature, and along with the case where the values of T_1 and T_2 exceeded the temperature T_* of water of maximum density, we studied the case where the temperature T_* was in the interval between T_1 and T_2.

In the English language literature, the flow considered is denoted by the term "lock exchange flow" [6]. Its Russian translation is lengthy. In similar problems of gas dynamics and hydraulics of open channels, it is customary to use the term "decay of an initial discontinuity" [8, 9]. In the case of a density discontinuity, the term "density jump" is employed [10]. Below, the hydrophysical effects due to the anomalous temperature dependence of water density will be called for brevity the anomalous effects.

The experiments were performed in a tank of width 20 cm and length 382 cm with the bottom and walls from Plexiglas 0.8 cm thick. The shield was placed asymmetrically with respect to the butt-ends of the tank. Preliminary experiment showed that such asymmetry has a significant effect on the liquid rise against gravity in large time intervals. The length of the left side of the tank separated by the shield was 158 cm, and that of the

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; bukreev@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 47, No. 1, pp. 66–73, January–February, 2006. Original article submitted January 21, 2005; revision submitted April 8, 2005.

right side was 224 cm. The butt-end walls of the tank were vertical. The shield was made of vinyl plastic 1.5 cm thick.

The lift velocity of the shield has a significant effect on the flow pattern. Below, we give the results of only those experiments in which the lower edge of the shield emerged from water in 1 ± 0.2 sec after the beginning of the lift. Since in the experiments, the difference in water density between the left and right sides of the tank was only a few tenths of percent, the difference between the free-surface levels on the right and left sides could have a strong effect. These levels were equalized by connecting the right and left sides with a 0.8 cm diameter tube. Before the removal of the shield, the tube was closed off.

The water temperature was decreased to 0.2°C using snow. Temperatures higher than room temperature were reached by the addition of hot water. On one of the sides of the tank, water was colored with ink, a standard aqueous solution of ink was added in an amount of less than 10^{-4} liters per 1 liter of water. The molecular diffusivity of ink in water was about 10^{-5} cm^2/sec. The initial water depth H was measured with a measuring needle with an error not worse than ± 0.05 cm. The water temperature was measured by a mercury thermometer and thermistor sensors with an absolute error not worse than 0.05°C. The density was determined from reference data at atmospheric pressure. The flow pattern was recorded by video cameras at a frequency of 25 frames/sec. The propagation speed of the leading edge of the perturbation, the characteristic depths of the layers, and the rise or immersion rates of the colored liquids were determined by processing on a computer. Each experiment was repeated not less than three times to decrease the error due to the effect of the lift velocity of the shield. The results of the repeated experiment were averaged.

Below, we give the results of two experiment with identical values of $H = 12.1$ cm and a water density difference between the left side (ρ_1) and right side (ρ_2) of the tank $\Delta\rho = \rho_1 - \rho_2 = 0.00206$ g/cm^3. In experiment No. 1, $T_1 = 15.1$°C and $T_2 = 25.1$°C. Both these values are larger than T_*; hence, there were no conditions for the development of the anomalous effects. In experiment No. 2, $T_1 = 0.4$°C and $T_2 = 21.9$°C; hence, there existed conditions for the development of the anomalous effects. The laboratory room temperature was 15.6°C in experiment No. 1 and 15.3°C in experiment 2.

In addition to the indicated parameters, the flow pattern is influenced by the molecular viscosity of water ν and the molecular diffusivity of heat χ. In interpreting the results of these experiments, it is necessary to take into account the possible effect of the molecular diffusivity of ink χ_1 and the heat fluxes through the free surface, bottom, and walls of the tank. The molecular viscosities and diffusivities depend on temperature.

In tanks of limited dimensions, the following little-known effect due to surface tension is observed: the molecular film at the interface between water and air adheres to the tank walls, and the smaller the dimensions of the tank in plan the stronger the adhesion. This effect is used, in particular, during blowing of soap bubbles or glass articles. A certain effort is required to detach the molecular film from the tank walls. In the experiments described here, with water flow velocities lower than 4 cm/sec, the film remained fixed and a viscous laminar boundary layer was formed on it as well as on the tank bottom. This effect influences the shape of the leading edge of the surface gravity current. In tanks of different dimensions, it can differ in some details.

A comparison of the results of experiment Nos. 1 and 2 shows that in the initial stage of the process with a duration of about 10 min, the effect of the anomalous temperature dependence of water density dominated over the effect of the other indicated weak factors. At large times, this anomaly of water also played an important role in the processes dependent on the motion prehistory.

Figures 1 and 2 illustrate the processes in the initial stage of motion. The time t is reckoned from the beginning of the lift of the shield. We call the upstream water the side of the tank to the left of the shield, where in the initial state, the water density was higher than its density on the right side. After the removal of the shield, a surface flat jet propagated upstream, and a bottom flat jet propagated downstream. In the absence of the anomalous effects (Fig. 1a and c), the course of the processes was similar to that described in [6]. In 3–4 sec after the beginning of motion of the shield, shear instability occurs (see Fig. 1a). Subsequently, a "head" characteristic of gravity currents of different origins formed at the leading edge of the jet (Fig. 1c). The shape of the "head" changed with time because of friction and mixing between the layers. Its sizes increased or decreased periodically. After the reflection from the butt-end walls of the tank, waves of the type of an undular bore propagated over the interface between the layers. The waves were smooth and completely damped in approximately 10 min.

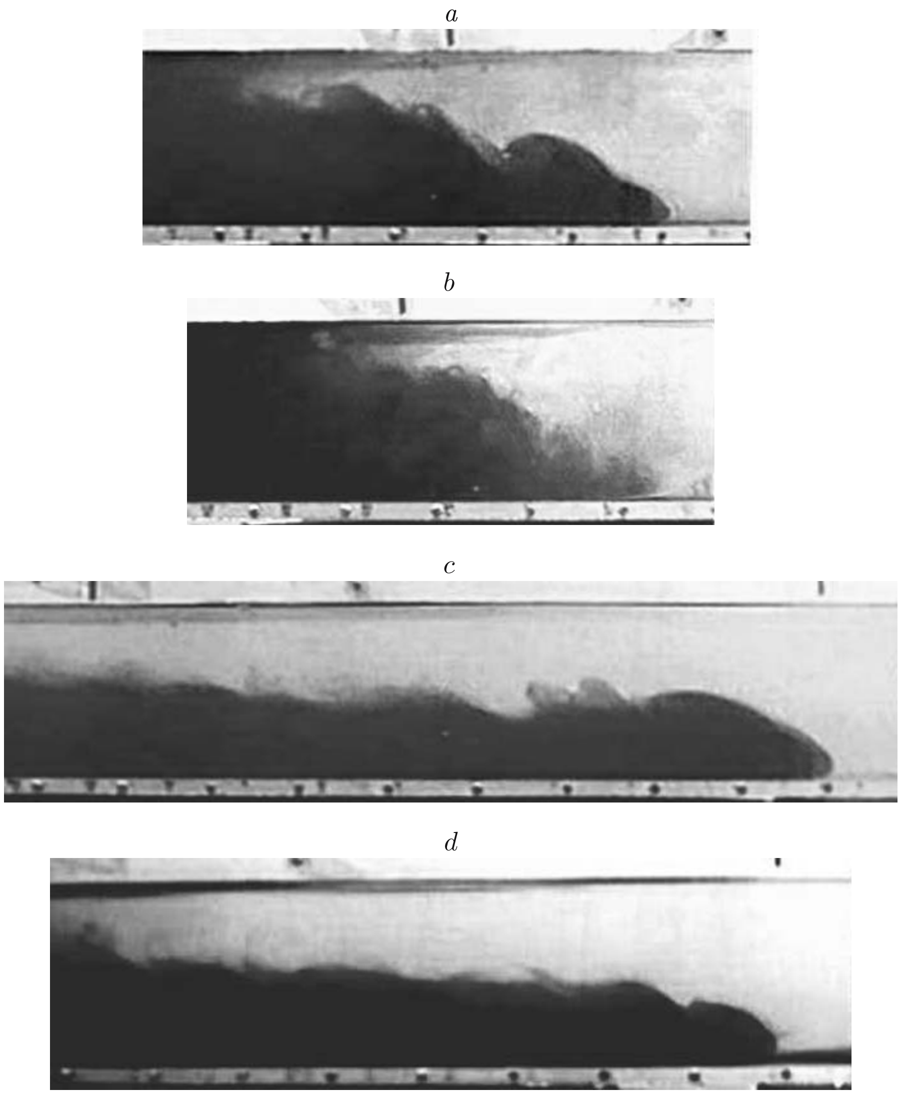

Fig. 1. Downstream gravity current: (a) experiment No. 1 at $t = 7$ sec; (b) experiment No. 2 at $t = 8$ sec; (c) experiment No. 1 at $t = 25$ sec; (d) experiment No. 2 at $t = 24$ sec.

In experiment No. 2, an unstable layer of maximum density formed after the removal of the shield. Its orientation was first predominantly vertical. This retarded the development of gravity current for approximately 2 sec compared to experiment No. 1. The pattern of motion instability in the following stages was also different because of a combined effect of the Kelvin–Helmholtz and Rayleigh–Taylor mechanisms. The process of stability loss was first more chaotic than in experiment No. 1 (Fig. 1b). Then, Rayleigh–Taylor instability began to dominate (Fig. 1d). In experiment No. 2, a distinct “head” formed approximately 15 sec later than in experiment No. 1.

The illustrations in Fig. 1 were obtained by coloring the heavier liquid with ink. The process of stability loss is better traced when the lighter liquid is colored. The corresponding illustrations from experiment No. 2 are given in Fig. 2. The structure shown in Fig. 2a formed upstream at about $2H$ from the leading edge of the perturbation. At this distance, Kelvin–Helmholtz instability dominated. With distance from the leading edge, the effect of Rayleigh–Taylor instability increased rapidly (Fig. 2b).

Figure 3 gives measured depths of the denser lower liquid $h_0^0(t^0)$ in the channel cross section where the shield was located. The superscript 0 indicates that the depth is normalized by H and time by the quantity $(H/\varepsilon g)^{0.5}$, where $\varepsilon = \Delta\rho/\rho_*$, ρ_* is the density at the temperature T_*, and g is the acceleration of gravity. From physical

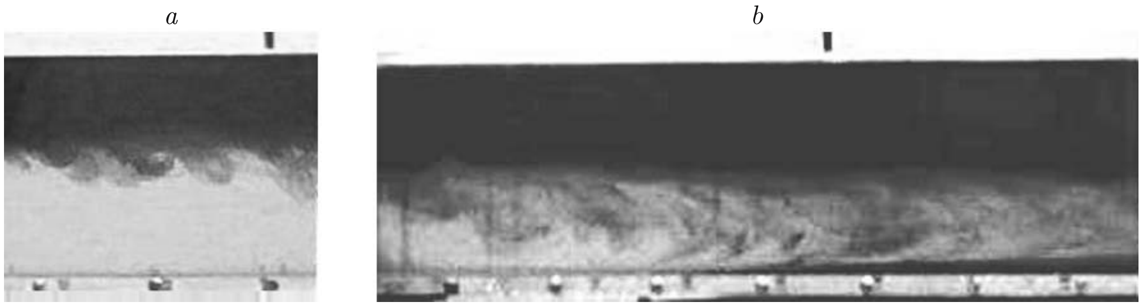

Fig. 2. Instability of motion in experiment No. 2 (upstream water; $t = 36$ sec) in the neighborhood of $x = -70$ cm (a) and $x = -3$ cm (b).

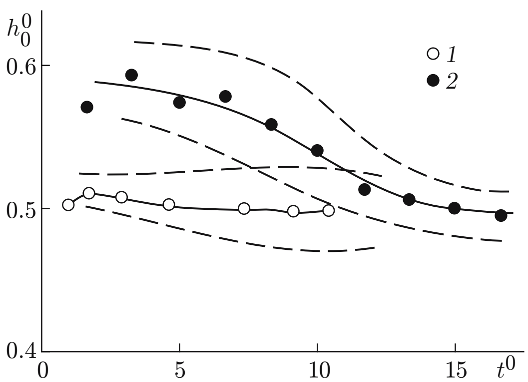

Fig. 3

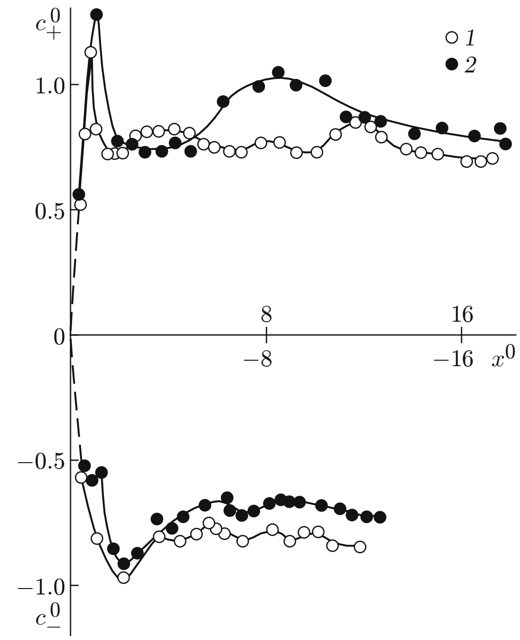

Fig. 4

Fig. 3. Depth of the lower layer at the shield location in experiment Nos. 1 (points 1) and 2 (points 2); the solid curves refer to the conditional interface between the main layers, and the dashed curves refer to the upper and lower boundaries of the mixed layer.

Fig. 4. Propagation speed of the leading edge of the perturbation in the downstream and upstream water in experiment Nos. 1 (points 1) and 2 (points 2).

considerations it can be expected that at a small density difference, $h_0^0 \to 0.5$ as t increases. This was confirmed in both experiments discussed here. A manifestation of the anomalous effects was that in experiment No. 2 this asymptotic value was reached markedly later than in experiment No. 1.

Figure 4 gives curves of the propagation speeds of the perturbation leading edges in the downstream (c_+^0) and upstream (c_-^0) water versus the longitudinal coordinate $x^0 = x/H$ reckoned from the initial position of the shield. A positive sign is adopted for the downstream water. The velocity is normalized by the quantity $(\varepsilon g H)^{0.5}$. The time dependence of the propagation speed has a complex nature. This is due not only to the nonstationarity of the process in the initial stage but also to the fact that because of mixing, the gravity current head alternatively accumulates and losses part of the liquid mass entering it.

The data of Fig. 4 show that the nonmonotonic temperature dependence of the water density has a marked effect on the propagation speed of the gravity current head. In experiment No. 2, the absolute value of this speed is

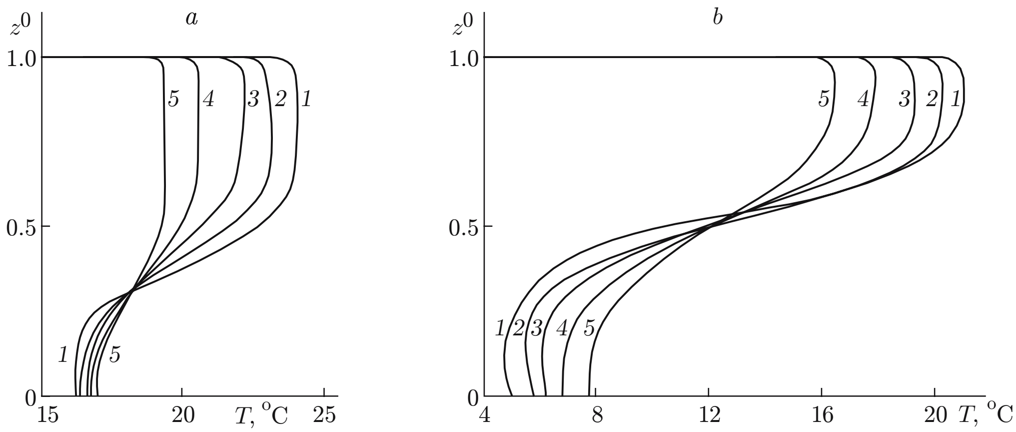

Fig. 5. Temperature profiles: (a) experiment No. 1 at $t = 10$ (1), 20 (2), 40 (3), and 70 (4), and 105 min (5); (b) experiment No. 2 at $t = 10$ (1), 20 (2), 30 (3), 40 (4), and 70 min (5).

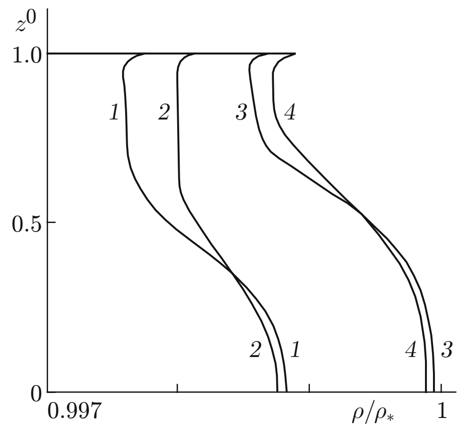

Fig. 6. Density profiles: curves 1 and 2 refer to experiment No. 1 at $t = 40$ and 105 min, respectively, and curves 3 and 4 refer to experiment No. 2 at $t = 40$ and 75 min, respectively.

higher in the downstream water and lower in the upstream water compared to experiment No. 1. This is explained by the fact that the formation of the layer of maximum density is accompanied by an increase in the potential energy of the liquid moving downstream. This example is of interest as an original method of converting thermal energy to mechanical energy.

Figure 5 gives temperature profiles at different times from the beginning of the lift of the shield, when the apparent motion has already stopped. The vertical coordinate z^0 is normalized by H. We note that the temperature profiles are intersected at almost the same point for a long time. The distance from the tank bottom to this point in experiment No. 2 is larger than that in experiment No. 1. In the time interval considered, the temperature difference in the upper and lower layers was also much larger in experiment No. 2 than in experiment No. 1. This implies that the anomalous effects reduce the rate of heat transfer between water in the tank and the laboratory room. The local temperature fall near the free surface in Fig. 5 is due to the phase transition of water to vapor.

Figure 6 gives the density profiles constructed from the temperature profiles. They also have a time independent point of intersection, which is higher in experiment No. 2 than in experiment 1. In addition, the profiles in experiment No. 2 are shifted toward higher density. Since the initial potential energy was identical in both experiments, this shift of the profiles implies that the potential energy of the system in experiment No. 2 increased compared to its value in experiment No. 1. A more detailed analysis shows that the potential energy in experiment No. 2 increased by approximately 0.1% even compared to its initial value (in the coordinate system with origin at the tank bottom). This is one more manifestation of the nonmonotonic temperature dependence of water density.

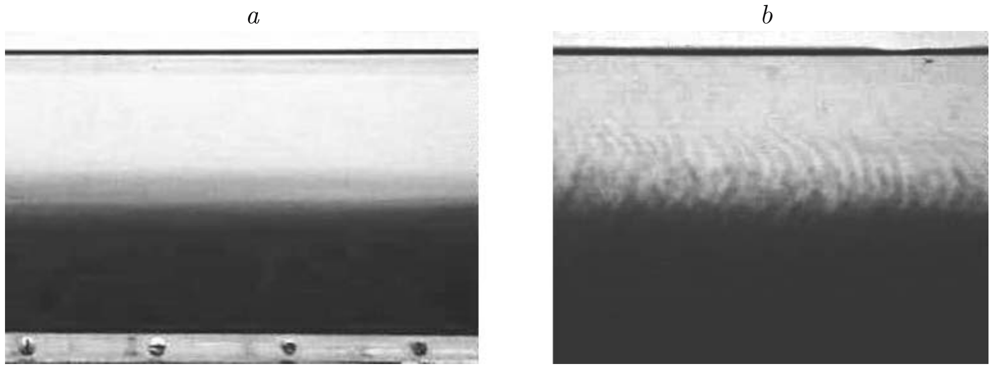

Fig. 7. Stratification in experiment No. 2 at $t = 12$ (a) and 60 min (b).

In approximately 10 min, the apparent motion ceased over the entire tank and a steady density stratification was established. Figure 7a gives a photograph taken in this stage of the process in experiment No. 2 with ink coloring of the heavier liquid. Then, in approximately 5 min, the peculiar secondary motion started again; the memory of it was stored in the ink distribution pattern given in Fig. 7b. First along the lateral walls and then over the entire width of the tank, the colored liquid rose against gravity in the form of discrete jets with mushroom-shaped structures in the head part. A similar process was also observed in experiment No. 1. In this experiment, unlike in experiment No. 2, the rise process begun earlier and the discrete jets were broader in the direction across the tank and more closely spaced in the x direction.

The possible reasons for the formation of the so-called fine structure of hydrophysical fields in the ocean are analyzed in detail in [11]. The picture in Fig. 7b is of interest because it contains memory of the fine-structure formation as a result of the rise of the denser liquid against gravity. The rise mechanism is due to the fact that the molecular diffusivities of heat χ and the dissolved additive χ_1 (ink in the present experiment) differ considerably in magnitude (χ_1 is two order of magnitude smaller than χ). In the description of this effect, the term double diffusion is used [10, 12]. The stratification of transparent and colored water shown in Fig. 7b may also be a result of the memory of the vortical nature of motion in the previous stages [5].

The stratification discussed here may be due to the heat transfer between water and the laboratory room through the free surface, lateral walls, and bottom of the tank. If the water and air temperatures are different, there is a local density change near the free surface due to vaporization and condensation. This was reflected in the temperature and density distributions with depth given above. In the density-stratified liquid, the heat flow through the lateral walls generates a peculiar convectional current in the form of so-called fingers [13]. These fingers are first extended in the horizontal direction. Then, due to the blocking effect [10], their vertical rise can begin [4].

Upon contact of cold water with the warmer tank bottom, conditions are produced for the formation of vertical jets due to Rayleigh–Taylor instability [14]. The anomalous temperature dependence of water density has a significant effect on this process, limiting the height to which the jets can rise. Water with maximum density is heated at the bottom, becomes lighter, and begins to rise in the form of jets. Subsequently, the jets again acquire the maximum density as a result of heat exchange with the surrounding water, and their rise ceases. This process may play an important role in deep pools such as lake Baikal.

In the experiments described here, the stratification of the colored and transparent water (see Fig. 7b) is due to the combined effect of all indicated factors except for double diffusion. The latter factor can lead to a rise of the heavy liquid if the temperature stratification plays a stabilizing role and the impurity stratification plays a destabilizing role (or vice versa) [11].

The picture given in Fig. 7b was not always observed. If, other conditions being equal, the shield was placed in the middle of the tank or was removed too rapidly (for less than 0.4 sec), then stratification similar to that shown in Fig. 7a remained until the complete equalization of the temperature over the entire tank. This suggests a significant effect of the nature of the initial vorticity. Rapid removal of the shield led to intense turbulent mixing, so that the initial size of the vortices and the time of their degeneration were smaller than those for slow removal. When the shield was shifted from the middle of the tank, the reflection of the gravitational jets from the right and left butt-end walls was not simultaneous, which also changed the nature of the vorticity.

In conclusion, it can be noted that in the theoretical analysis of the examined current using the potential fluid flow model, it is assumed that the perturbation propagates at a constant speed [6, 7]. The data of Fig. 5 show that this speed is not constant. The variation in the propagation speed of the gravity current results not only from the energy losses due to friction but also from the mass and momentum transfer between the main layers during mixing. A theoretical analysis of gravity currents taking into account mixing between the layers but ignoring the anomalous effects was performed in [15].

We thank E. M. Romanov and N. V. Gavrilov for assistance in the experiments.

This work was supported by the Russian Foundation for Basic Research (Grant No. 04-01-00040), the foundation "Leading Scientific Schools of Russia" (Grant No. NSh-22-2003.5, and the Integration Project of the Russian Academy of Sciences "Hydrodynamics of Baikal Water."

REFERENCES

1. S. S. Zilitinkevich, K. D. Reiman, and A. Yu. Terzhevik, "The thermal bar," *J. Fluid Mech.*, **236**, 27–42 (1992).
2. P. R. Holland and A. A. Kay, "Review of the physics and ecological implications of the thermal bar circulation," *Limnologica*, **33**, No. 3, 153–162 (2003).
3. V. I. Bukreev, "Hydrodynamic instability conditioned by anomalous relation of water density from temperature," *Dokl. Ross. Akad. Nauk*, **396**, No. 6, 770–772 (2004).
4. V. I. Bukreev, "Effect of the anomalous temperature dependence of water density on surface gravity current," *J. Appl. Mech. Tech. Phys.*, **46**, No. 1, 49–54 (2005).
5. V. I. Bukreev, "Effect of the anomalous temperature dependence of water density on circular jet propagation," *Fluid Dyn.*, **40**, No. 2, 202–208 (2005).
6. J. E. Simpson, *Gravity Currents in the Environment and the Laboratory*, Cambridge Univ. Press, Cambridge (1997).
7. T. B. Benjamin, "Gravity currents and related phenomena," *J. Fluid Mech.*, **31**, Part 2, 209–243 (1968).
8. L. D. Landau and E. M. Lifshits, *Mechanics of Continua* [in Russian], Moscow, Gostekhteoretizdat (1954).
9. A. A. Atavin, M. T. Gladyshev, and S. M. Shugrin, "On discontinuous flows in open channels," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], No. 22, Inst. of Hydrodynamics, Sib. Div., Acad. of Sci. of the USSR, Novosibirsk (1975), pp. 37–64.
10. J. S. Turner, *Buoyancy Effects in Fluids*, Cambridge Univ. Press, Cambridge (1973).
11. K. N. Fedorov, "Fine structure of hydrophysical fields in the ocean," in: *Oceanology. Physics of Ocean*, Vol. 1: *Hydrophysics of Ocean* [in Russian], Nauka, Moscow (1978), pp. 113–147.
12. A. W.-K. Law, W. F. Ho, and S. G. Monismith, "Double diffusive effect on desalination discharges," *J. Hydraul. Eng.*, **130**, No. 5, 450–457 (2004).
13. V. V. Levitskii and Yu. D. Chashechkin, "Thermoconcentration convection under homogeneous lateral heating," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 5, 112–124 (1995).
14. J. W. Elder, "The unstable thermal interface," *J. Fluid Mech.*, **32**, Part 1, 69–96 (1968).
15. V. Yu. Liapidevskii and V. M. Teshukov, *Mathematical Models for the Propagation of Long Waves in an Inhomogeneous Liquid* [in Russian], Izd. Sib. Otd. Akad. Nauk, Novosibirsk (2000).